# **Integrating Stress Enhanced Recovery into a Work Hardening Model**

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This study reports on the development of a dislocation based model and its application to recovery. The model calculates the flow stress and dislocation density evolution during annealing. In order to validate the model, the recovery behavior of a commercial aluminum alloy was investigated by means of three alternative experimental methods: stress relaxation (SR), double tension tests with load (DTL) and unloaded (DTU). The experimental results revealed that external stresses have a significant impact on the recovery processes. Accordingly, external stresses are considered in the model, which enables to distinguish between stress-free and stress-enhanced recovery. In the current contribution, a respective dislocation based model is introduced.

Keywords: Recovery, Stress relaxation test, Double tension test, Modeling.

# 1. Introduction

During aluminum sheet production, softening processes including recovery and recrystallization can significantly influence materials properties, e.g. the strength. Accordingly, the modeling of these softening processes is of great scientific and technological relevance. Moreover, a successful simulation of recrystallization is strongly dependent on a reliable and precise understanding and prediction of the recovery processes [1, 2]. Hence, recovery must be modeled and calibrated accurately. In the current study, the recovery mechanism and its kinetics were investigated by three alternative experimental methods: stress relaxation (SR) [3-11] and double tension tests with external load (DTL) [9-11] and unloaded (DTU) [9-13]. In the literature, standard annealing tests, as e.g. DTU tests, have been widely used do derive recovery kinetics [12-14]. These standard methods are generally time consuming and cost-intensive. The aim of the present study was to replace these standard techniques by stress relaxation tests since they do require numerous test series. Therefore, an extensive study of the obtained stress evolution was done in order to understand the underlying mechanisms [9-11]. The gained information was implemented in a dislocation density based recovery model. This model predicts the stress evolution of SR, DTL and DTU tests and its temperature dependence with a single set of seven fit parameters. This parameter set, in turn, can essentially be gained from SR tests alone.

## 2. Evaluation of recovery kinetics using various experimental approaches

The evaluation of recovery kinetics was discussed by using the stress evolution obtained from SR, DTL and DTU tests. The aim of the current investigations was to establish stress relaxation (SR) tests as a method for determining recovery kinetics. Even though SR tests are relatively simple in execution and well documented in the literature [3-8], this method has not been investigated systematically for deriving recovery kinetics, in spite of the relatively low cost involved. The applicability of SR tests was verified by double tension tests with load (DTL) and without load (DTU) since DT tests, usually without load, are already established as a proper test for determination of softening kinetics [12, 13]. All tests were performed on a commercial aluminum alloy 3103 and carried out in an electromechanical testing machine. They were conducted in the temperature range  $175-225^{\circ}$ C at a constant true strain rate of  $7 \cdot 10^{-4} \text{s}^{-1}$ . Details on this material, specimen preparation and on experimental techniques were reported recently [9-11].



Fig.1: Outline of the applied techniques; (a) conventional tensile test (TT), (b) stress relaxation test (SR), (c) double tension test with load (DTL) and (d) double tension test without load (DTU).



Fig.2: Stress evolution vs. time of the three alternative experimental methods; DTU, DTL and SR at a temperature of 200°C.

The applied techniques are schematically shown in Fig.1. The conventional tensile test TT (a) is used to define the reference state with the initial flow stress  $\sigma_0$ . However, deformation at various elevated temperatures up to a predefined strain generally causes microstructural differences. In order to avoid such microstructural differences at  $\sigma_0$ , a refined experimental procedure was done [9-11]. For each temperature T = 175°C, 200°C and 225°C to be used later on for recovery, certain strain values  $\varepsilon_0(T) = \varepsilon(t = 0, T)$  for the first deformation step were determined in preparatory experiments. These values  $\varepsilon_0(T)$  were determined such that after the deformation at temperature T and immediate quenching to room temperature RT, a subsequent deformation at RT yielded almost identical yield stress and secondary flow curves. This means, that for each temperature T a different strain value  $\varepsilon_0$ was required. For the recovery tests at temperatures T, the pre-deformation was performed, also at T, up to the corresponding values  $\varepsilon_0$ . Hence, all specimens had virtually the same microstructure and initial flow stress  $\sigma_0$  at the start of the recovery tests.

The subsequent procedures for SR, DTL and DTU (Figs. 1b, c, d) differed slightly. In the case of SR tests, the crosshead of the machine was arrested for a relaxation time of 5 h. The stress and time were continuously recorded during this period. This test needs to be performed only once for each temperature. The DTL tests were, in principle, SR tests with reloading after defined relaxation times of t = 60 s, ...., 5 h. During DTU tests the specimen was, in contrast to DTL, completely unloaded during the holding times, and subsequently reloaded. The yield stress dependent on relaxation time was obtained from the reloading curves of DTL and DTU by extrapolation of the elastic and back-extrapolation of the plastic regime [9-11]. The DTL and DTU tests require, obviously, new measurements with new samples for each time period (and each temperature).

Fig. 2 shows the stress evolution over time obtained from DTU, DTL and SR tests. It is exemplarily shown for T = 200°C. All tests of the current study yielded similar observations, irrespective of temperature. Obviously, the characteristics of the stress decays differ significantly. The stress decrease during SR was much stronger compared to the yield stress drops obtained from both DT tests. The yield stress drops of DTL and DTU tests can be attributed due to the occurrence of recovery, regardless of the loading state. In the case of DTU tests, the stress decrease was caused by static recovery. By contrast, the stronger stress decrease obtained from DTL tests was caused due to enhancement of the recovery processes by the applied external stress. This, in turn, implies that stress-dependent recovery processes occur during the relaxation period [5, 9-11, 15]. The stress decay recorded during the SR tests involves the same recovery as the DTL tests since the specimens were treated equally. But beside recovery the stronger stress decay of SR was caused by continued slip at

the expense of the elastic strain. This difference between DTL and SR tests was already reported by other authors [3-8].

Recently, it was shown that one set of stress relaxation curves at varying temperatures enables to predict accurately the stress enhanced recovery kinetics, as measured from DTL tests [11]. Details on the equations [11, 15, 18, 19] and the fitting procedure [11] were published elsewhere. It was demonstrated that SR tests deliver the same information on recovery kinetics as the vastly more laborious DTL tests [11]. But the prediction of the static recovery kinetics, as delivered by DTU tests, failed by that approach. Therefore, the prediction of the recovery kinetics – stress free and enhanced – is discussed here in terms of a physics based recovery model.

#### 3. Modeling recovery kinetics

The objective of this work was to develop a dislocation density based model to depict the stress evolution during stress relaxation. In addition, the stress influence on the structure evolution during recovery - DTL as well as DTU - should be predicted. This is done on the basis of a physical description  $\sigma = \sigma(\varepsilon, \rho, T)$ , where the dislocation density  $\rho$  represents a true state variable. The total dislocation density  $\rho_{\text{total}}$  consists of three types of dislocations as internal state variables, namely geometrically necessary dislocation  $\rho_{\text{GND}}$ , mobile dislocation density  $\rho_{\text{m}}$ , and dipole dislocation density  $\rho_{\text{dip}}$ .

The dislocation type  $\rho_{GND}$  accommodates the crystal lattice rotation. It is assumed to be constant (no recovery) and is used as a fit parameter. The second dislocation type  $\rho_m$  covers all mobile dislocations, they are involved in the kinetic equation of state: [20]

$$\dot{\varepsilon}_{pl} = \overline{M}^{-1} \cdot \rho_m \cdot b \cdot \upsilon(\tau_{eff}, T) \tag{1}$$

where  $\dot{\varepsilon}_{pl}$  is the plastic strain rate,  $\overline{M} = 3.06$  is the average Taylor factor for uniaxial tension, b = 0.286 nm is the Burgers vector of aluminum and  $\upsilon(\tau_{eff}, T)$  is the average glide velocity depending on the effective stress  $\tau_{eff}$  and the temperature T. During the relaxation period the crosshead is arrested so that the total strain rate vanishes. Hence  $\dot{\varepsilon}_{el} = -\dot{\varepsilon}_{pl}$ , where  $\dot{\varepsilon}_{el}$  is the elastic strain rate. Since mobile dislocations continue to move after the crosshead is arrested, they cause a small bit of inelastic strain  $\dot{\varepsilon}_{pl}$  at the expense of the elastic strain. The corresponding stress decay is [21]:

$$\dot{\sigma} = -E_s \cdot \dot{\varepsilon}_{pl} \tag{2}$$

where  $E_s$  is the effective elastic modulus of the system machine-sample. Altogether, the flow stress decreases due to the plastic elongation  $\varepsilon_{pl}$  of the sample. For the function  $\upsilon(\tau_{eff}, T)$ , thermally activated slip is assumed as described in [20] with an activation energy  $Q_{slip}$  and activation volume  $V_{slip}$ . The effective stress  $\tau_{eff}$  is taken to be  $\tau_{eff} = \tau_{ext} - \tau_{int} - \tau_{part}$ , where  $\tau_{ext}$  denotes the external applied stress,  $\tau_{int}$  the mean internal stress caused by dislocations, and  $\tau_{part}$  the particle strengthening contribution.

It is known that during relaxation at elevated temperatures, the dislocation density is reduced due to recovery processes [15]. In this study, it is assumed that the dipole dislocation density  $\rho_{dip}$  decreases by climb [22] according to

$$\dot{\rho}_{dip} = \mathbf{V}_{\text{climb}} \cdot d_{dip}^{-1} \cdot \rho_{dip} \tag{3}$$

where  $v_{climb}$  is the mean climb velocity and  $d_{dip}$  is the dipole height. This dipole height  $d_{dip}$  was chosen 1.6 nm, which was already determined from atomic-scale numerical simulations by Aslanides and Pontikis [23]. In the current study,  $v_{climb}$  is composed by three contributions, all of which are assumed to be assisted by internal stresses from dislocations in the material:

$$v_{\text{climb}} = (4 \cdot v^0 + v^+ + v^-) / 6.$$
(4)

The idea here is that for most recovering dipoles (four ones out of six assumed here), the tensor components of the external stress do not match those of the internal stresses, which are responsible for the assistance of recovery. Hence, the climb velocity contribution  $v^0$  of this case is independent of the external stress  $\tau_{ext}$ :

$$v^{0} = b \cdot v_{D} \cdot \exp\left(-\frac{Q_{\text{climb}} - (\tau_{\text{int}} \cdot F_{\text{struct}}) \cdot V_{\text{climb}}}{kT}\right)$$
(5)

where  $v_D = 10^{13} s^{-1}$  is the Debye frequency,  $Q_{climb}$  is the activation energy, k is the Boltzmann factor,  $V_{climb}$  is the activation volume for climb, and  $F_{struct} = 10$  is a structure factor. The product of  $\tau_{int} \cdot F_{struct}$  depicts the internal stresses in the wall, whereby  $\tau_{int}$  is the work hardening part overall. In other words,  $F_{struct}$  defines the concentration of internal stresses in the walls resulting from the high dislocation density in the walls.  $F_{struct}$  is discussed later on because it is essential for the distinction of DTU and DTL experiments. The internal stress is given by  $\tau_{int} = \alpha \cdot b \cdot G \cdot \sqrt{\rho_{total}}$  with  $\alpha = 0.5$  as geometrical constant, G the shear modulus, and  $\rho_{total} = \rho_{GND} + \rho_{dip} + \rho_{mobile}$ .

Unlike the cases covered by Eq.5, the remaining recovery events are either accelerated or decelerated by the external stress  $\tau_{ext}$ , depending on the local sign relation between  $\tau_{ext}$  and  $\tau_{int}$ . In Eq.4 it is assumed that these cases, described by  $v^+$  and  $v^-$ , respectively, occur equally frequent. Moreover, it is assumed that the thermally activated recovery event is the same one as assumed for Eq. 5. Hence,

$$\mathbf{v}^{+\prime-} = b \cdot \mathbf{v}_{D} \cdot \exp\left(-\frac{\mathbf{Q}_{\mathsf{climb}} - \left(\tau_{\mathsf{int}} \cdot F_{\mathsf{struct}} \pm \tau_{\mathsf{ext}}\right) \cdot \mathbf{V}_{\mathsf{climb}}}{kT}\right)$$
(6)

where  $Q_{\text{climb}}$ ,  $V_{\text{climb}}$ , and  $F_{\text{struct}}$  are the same values as in Eq.5. By this approach, acceleration and deceleration of recovery by external stresses cancel each other out at first order. Altogether with this model, the stress evolution of SR tests can be simulated with one single set of seven fit parameters for a range of temperatures:  $\sigma_{SR}(t) = \overline{M} \cdot \tau_{ext}(t)$ . In the same set of simulations, the stress evolution of DTL tests is described by  $\sigma_{DTL}(t) = \overline{M} \cdot (\tau_{int}(t) + \tau_{part} + \tau_{eff})$ . In addition, the stress evolution of DTU tests can be calculated by repeating the simulations with the same parameter set, but inserting zero for  $\tau_{ext}$  in Eq.6. Then,  $\sigma_{DTU}(t) = \overline{M} \cdot (\tau_{int}(t, \tau_{ext} = 0) + \tau_{part} + \tau_{eff})$ .

In order to validate the model, the stress evolution of SR tests was fitted in the temperature range 175-225°C. The obtained evolution of the dislocation density  $\rho_{total}$  was inserted in the Equation for  $\sigma_{DTL}$  and  $\sigma_{DTU}$ , mentioned above. Investigations suggested earlier [11] have shown that the evaluation was refined by combining the SR evaluation with a single DTL measurement taken directly after the relaxation: by reloading the SR specimen. Hence, one DTL value could be gained from the same specimen without much additional time or work. A combined evaluation employed both SR and DTL calculations, and optimized all seven parameters with chosen weights to error allowances for SR and DTL. In Fig.3 the experimental and modeled stress evolution of the SR tests in the temperature range of 175-225°C are shown. The seven fit parameters were allowed in a

reasonable range, and each was constant for all temperatures. An exception to this is the activation volume for slip  $V_{slip}$ : it was necessary for reasonable fits to allow for an increase of  $V_{slip}$  with increasing temperature. In order to interpret this, further experimental investigations concerning a varying solute content are planned. It is not considered important here, though, since  $V_{slip}$  is no recovery parameter.

But obviously, the modeled (black) stress evolution of SR can predict rather well the experimental curves (grey) (Fig.3). Furthermore, the stress decay of DTU (line/open symbols) and DTL (line/filled symbols) tests was calculated as indicated above; the evolutions of  $\sigma_{DTL}$  and  $\sigma_{DTU}$  are compared to the experimentally determined stress values (Fig.4). Using this model the prediction of stress-free and stress enhanced recovery kinetics is quite good in light of the experimental scatter. However, there is still room for improvement by future model refinements. One deficiency of the prediction concerns the difference between  $\sigma_{DTL}$  and  $\sigma_{DTU}$ . This difference is controlled by the structure parameter  $F_{struct}$ , which had been fixed here at  $F_{struct} = 10$ . With larger values of  $F_{struct}$ , and smaller values of  $V_{climb}$  such that  $F_{struct} * V_{climb}$  is kept constant, the influence of  $\tau_{ext}$  and hence the difference between  $\sigma_{DTL}$  and  $\sigma_{DTU}$  decreases (Eq.6). At any constant value of  $F_{struct}$ , however, the difference  $\sigma_{DTU}$ - $\sigma_{DTL}$  increases with decreasing temperature, whereas the experimental rather show the opposite tendency (earlier experimental works showed no tendency at all [11]). Therefore, model refinements are needed in future.



Fig.3: Experimental (grey) and modeled (black) stress evolution of SR tests in the temperature range of 175-225°C.



Fig.4: DTU (grey line) and DTL (black line) stress evolution derived from SR tests compared to direct DTU (open symbols) and DTL (filled symbols) experiments.

#### 4. Summary and conclusion

Stress relaxation (SR) tests and double tension tests with (DTL) and without load (DTU) were performed to measure the recovery kinetics of an aluminum alloy 3103 for a range of temperatures. A model was developed that is able to describe the three different stress evolutions over time for all temperatures with a single set of seven parameters. With this model used for evaluation, it is possible to derive the recovery kinetics of materials from only one SR test for each temperature used. Hence, SR tests can replace the vastly more laborious DTL or DTU tests. Moreover, it is important to note that essentially, the new model is able to capture the impact of external stresses on recovery kinetics. This will be very valuable for future refinements of statistical flow stress models like 3IVM+ [20], which so far involve much less accurate approaches for recovery.

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