

Development of Three-dimensional Plastic Strain Mapping Technique for X-ray micro-tomography

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The synchrotron micro-tomography can reveal the existence of pores, particles and cracks in non-destructive in approximately 1 μm resolution. In this study, three-dimensional mapping technique has been developed to observe inhomogeneous plastic deformation affected by metallurgical microstructures within a small specimen. Measuring displacement of microstructural features like particles and pores brings us local strain within the specimen. However, it is difficult to track these markers position correctly without errors. Marker-tracking method with predicting marker position was developed to obtain the strain map in inhomogeneously and highly deformed region within the specimen. However, it was impossible to evaluate the tracking successful ratio in the developed method because we cannot obtain the correct matching pairs before and after material deformation. Therefore, simulation model were developed and utilized for evaluating the tracking success ratio. The model that simulates tensile test in an aluminum alloy was built with taking actual deformation pattern into consideration. It was found that the developed method showed good performance by evaluating the successful tracking ratio and tracked marker ratio. The prediction of marker position after deformation was very effective in tensile deformation with necking. Almost 100% ratios of successful tracking and tracked marker were indicated not only in homogeneous deformation up to 10% strain but also in inhomogeneous deformation up to 5% strain.

Keywords: *micro-tomography, plastic strain, local deformation, tracking method, three-dimension*

1. Introduction

Recently, X-ray micro-tomography has been utilized for characterization of deformation and fracture behaviors in aluminum alloys. The synchrotron micro-tomography can reveal the existence of pores, particles and cracks in non-destructive in approximately 1 micrometer resolution [1,2]. Our research group has developed three-dimensional strain mapping method by means of microstructural features tracking in the X-ray micro-tomography [3,4]. It is expected that very high-density strain mapping is found because huge number of microstructural features are observed in the synchrotron micro-tomography. However, there is a risk that a tracking error causes incorrect strain map. The confidence of successful tracking is important for the developed method. The error tracking produces wrong strain concentration in the strain map. In case of studies about deformation and fracture, the strain concentration is important and interest matter for us. Therefore, development of error free tracking method is necessary.

To evaluate the tracking successful ratio allows verifying the tracking performance in the developed method. However, evaluation of the tracking successful ratio is not feasible, because number of markers is too many to confirm by humans eye. In the previous study, the marker tracking method using the matching probability parameter (MPP) has been developed for three-dimensional strain mapping. The ratio of tracked marker and the ratio of successful tracking have been evaluated in case of about two hundreds numbers of markers. Further, the calculation method of

three-dimensional strain mapping from the tracking results was developed and shown [4]. However, the previous tracking method had a problem that the ratio of successful tracking was low in situation of inhomogeneous deformation. The ratios were only evaluated in the limited number of marker. To solve this two problem is attempted in this study. Namely, new tracking method applying a trajectory prediction was developed to increase tracking successful ratio in the situation of inhomogeneous deformation. Moreover, simulation model of a tensile deformation in aluminum alloy, which contains equivalent numbers of marker in an actual aluminum alloy, was prepared to evaluate the tracking successful ratio in new developed method. The new method shows high successful tracking ratio in the simulated deformation model. It is reliable in actual data analysis.

2. Trajectory prediction

Macroscopic trajectory prediction (MTP), which moves marker position assuming the homogeneous deformation pattern with the Poisson's ratio of 0.5, has been developed in the previous study to compensate marker displacements before and after deformation. In the situation developing inhomogeneous deformation like a necking in tensile test, some tracking error was confirmed. It was difficult to predict deformation pattern in the whole area of specimen. In this study, trajectory prediction for each marker was applied for compensating marker displacements before and after deformation. Figure 1 shows schematic illustration of the trajectory prediction. Polynomial extrapolation was used to predict marker positions in next step. The extrapolation polynomial is in the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0. \tag{1}$$

If we use the marker positions during the deformation, we get a system of linear equations in the coefficients a_k .

$$\begin{bmatrix} x_0^n & x_0^{n-1} & x_0^{n-2} & \dots & x_0 & 1 \\ x_1^n & x_1^{n-1} & x_1^{n-2} & \dots & x_1 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ x_n^n & x_n^{n-1} & x_n^{n-2} & \dots & x_n & 1 \end{bmatrix} \begin{bmatrix} a_n \\ a_{n-1} \\ \vdots \\ a_0 \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_n \end{bmatrix} \tag{2}$$

The system for a_k can be solved to construct the extrapolation function of $p(x)$.

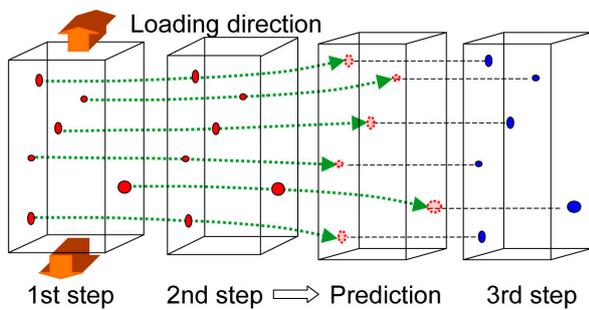


Fig. 1 Schematic illustration of trajectory prediction.

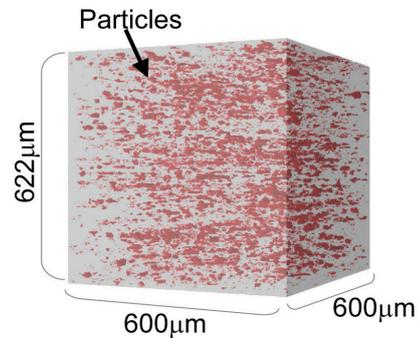


Fig. 2 Marker particles in 2024 aluminum alloy obtained synchrotron micro-tomography. The particles were utilized for the simulated deformation model.

3. Simulation model to evaluate the tracking method

Simulated deformation model has been prepared to evaluate tracking performance of developed method in the condition of large numbers of marker. The marker size distribution, macroscopic deformation pattern and microscopic deformation were considered to reproduce actual deformation in the model.

3.1 Markers

Virtual sample for the simulated deformation model is aluminum alloy, which has been often used for deformation and fracture studies by using the micro-tomography. The size data of Al₂Cu particles in a 2024 aluminum alloy scanned by the micro-tomography were utilized as markers in the simulated deformation model. The virtual specimen size was given as 600×600×650 μm³ which corresponds to the field of view (FOV) in the synchrotron micro-tomography with a (0.5 μm)³ voxel size. The particles size data were prepared from tomographic volume image by using three-dimensional image processing. Threshold value of linear absorption coefficient, 60 cm⁻¹ was used to separate the aluminum matrix and the Al₂Cu particles in the tomographic volume image. The particles with a size greater than 28 voxels were used. 103611 particles existed in 1 mm³ in the 2024 aluminum alloy. Figure 2 shows the actual particles distribution obtained by three-dimensional image reconstruction after tomographic scan. The average diameter of particles was 5.2 μm. Considering the particles density in the aluminum alloy, 24636 marker particles were picked up from the actual size data and placed randomly in FOV space of 600×600×650 μm³ as an initial condition.

During the deformation, marker size will change slightly due to measurement errors in connection with digital image analysis. The changes were considered and simulated in this study. The error of marker volume was simulated with the following equations,

$$V' = \frac{4}{3} \pi (r + \Delta r)^3, \quad (3)$$

where V' is a simulated volume, r and Δr are given by radius of particles and normal distribution function with standard deviation, respectively. By the similar way, the error of marker surface area was also simulated as follows,

$$S' = 4\pi (r + \Delta r)^2. \quad (4)$$

3.2 Macroscopic deformation

Macroscopic deformation was simply given to the model assuming Z-axis (loading axis) symmetrical deformation, because actual deformation pattern is complex and depends on materials properties, specimen shapes and so on. The deformation was handled on a cylindrical coordinate represented by R , θ and Z . We gave strain distribution in the R -direction as a function of the Z -position in the deformation model. The strain distribution of R -direction depending on the Z -position, $\varepsilon_r(z)$ gives displacement of the markers in the R -direction after the deformation assuming a constant strain in R -axis. The displacement of markers in Z -axis was given through the volume constant condition. The strain function, $\varepsilon_r(z)$ for the macroscopic deformation was given by

$$\varepsilon_r(z) = a \times \exp\left(\frac{-(z-b)^2}{2(c/3)^2}\right), \quad (5)$$

where a , b and c are arbitrary constants for changing deformation pattern. Although there is little physical meaning, the Gaussian type function was selected and used conveniently for simulating tensile deformation pattern. It is not so easy to describe the strain function with a few parameters. Tensile deformation was supposed in the model. The nominal stress and nominal strain curve is shown in Fig. 3. The virtual specimen deforms homogeneously up to 10%. After that, inhomogeneous plastic deformation was developed with necking as shown in Fig. 3.

3.2 Microscopic deformation

Not only effects of the macroscopic deformation but also effects of microscopic deformation have been considered in this study. The microscopic deformation, which is deformation of crystallographic

grain, was given as simple shear deformation and rigid rotation. Firstly, the markers were segmented into the groups that corresponds with virtual grains with a diameter of 100~150 μm . Next, crystallographic orientation was given randomly to the virtual grains. The shear deformation for the individual grains was carried out simply in a slip system having the smallest Schmid factor. The rotation manner was defined in accordance with the reference [5]. Namely, grain rotates depending on crystallographic orientation. The manner is shown as follows: (1) when the region where misorientation from $\langle 100 \rangle$ is less than 24.2 degree, grain rotates toward $\langle 100 \rangle$, (2) when the region where misorientation from $\langle 221 \rangle$ is less than 8 degree, grain rotates toward $\langle 111 \rangle$, (3) when the region where misorientation from $\langle 110 \rangle$ is less than 24.0 degree, grain rotates toward a line connecting with $\langle 100 \rangle$ and $\langle 111 \rangle$. (4) In case of the others, grain rotates toward $\langle 111 \rangle$ along a line connecting with $\langle 100 \rangle$ and $\langle 111 \rangle$. We assumed the magnitude of shear deformation and rotation rise in proportion to $\varepsilon_r(z)$.

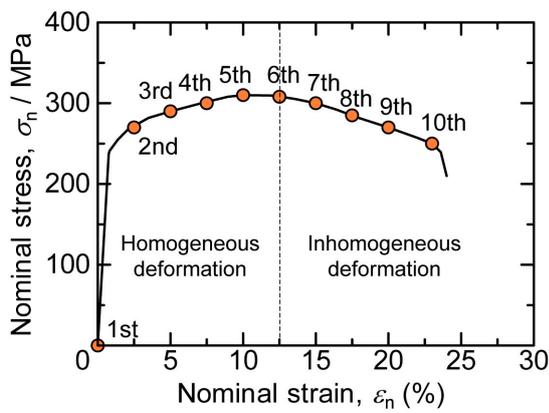


Fig. 3 Supposed nominal stress and nominal strain curve in the model, which is tensile deformation in aluminum alloy.

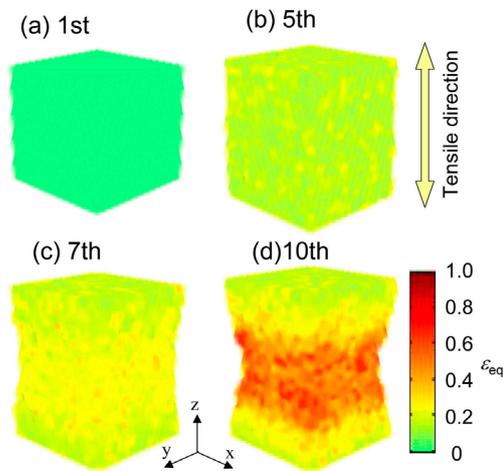


Fig. 4 Changes in equivalent strain distribution in the simulated deformation model in tensile.

Figure 4 shows three-dimensional distributions of the equivalent strain in the simulated tensile-deformation model. It is found that homogeneous strain develops gradually in the Fig. 4 (a) and (b). Microscopic deformation is observed as uneven color in the figure. Necking occurs in the center of specimen in Fig. 4 (c) and (d). The strain develops intensively in the part of necking.

4. Result of Tracking

The tracking performance was evaluated with two indexes. The first one is a ratio of tracked marker, ρ and another is a ratio of successful tracking, η . Ratio of tracked marker, ρ is given as

$$\rho = \frac{\text{No. of tracked marker}}{\text{No. of total marker}} \times 100(\%). \quad (6)$$

Note that number of total marker after deformation was utilized as standard in case of the tensile test. Ratio of successful tracking, η is given as

$$\eta = \frac{\text{No. of successful tracked marker}}{\text{No. of tracked marker}} \times 100(\%). \quad (7)$$

Figure 5 indicates (a) ratio of tracked marker and (b) ratio of successful tracking in each deformation step. The previous MTP method and the new developed trajectory prediction method applied 1st-, 2nd- and 3rd-degree polynomial extrapolation were evaluated with the simulated deformation model. The tracking was carried out in due order of deformation step. The ratio of tracked marker and the ratio of successful tracking show 100% from 2.5% to 12.5% strains without dependence of the tracking method. This is because the model deformed homogeneously. Inhomogeneous deformation began after 12.5% strain. The ratio of tracked marker decreases after

12.5% strain. The drop is remarkable in the MTP method. The trajectory prediction method with higher-degree polynomial extrapolation makes the ratio of tracked marker high. However, at 15% strain the ratio drops. This is reason that the deformation pattern shifted from homogeneous deformation to inhomogeneous deformation at 15% strain. Concerning with the ratio of successful tracking, the trajectory prediction method keeps high ratio while the ratio of successful tracking in the MTP method decrease gradually. The trajectory prediction method makes tracking possible in inhomogeneous deformation.

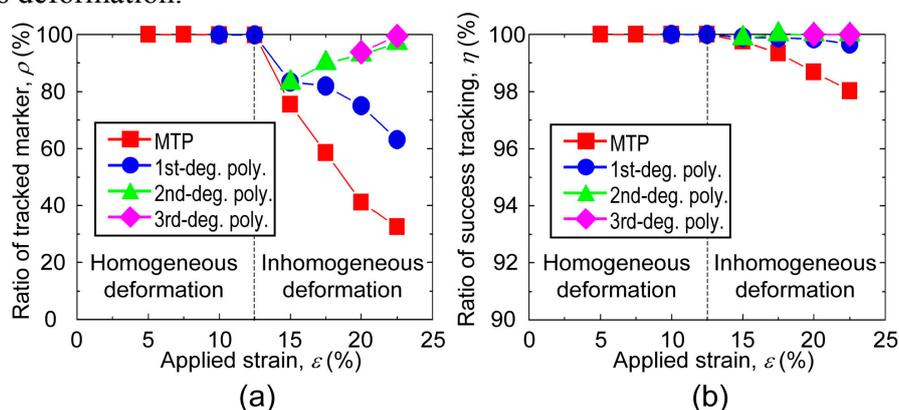


Fig. 5 Evaluation of tracking performance in each deformation step; (a) ratio of tracked marker and (b) ratio of successful tracking.

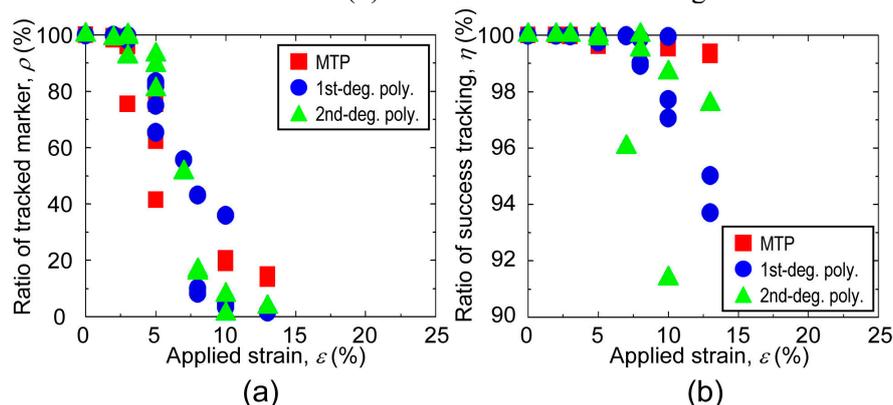


Fig. 6 Evaluation of tracking performance in different deformation situation; (a) ratio of tracked marker and (b) ratio of successful tracking

The tracking results in various combinations of strain step are shown in Fig. 6. The different combination of strain step means different deformation situation. For example, the deformation of 5% strain by the combination of 6th-step and 8th-step is quite different to that of 5% strain by the combination of 7th-step and 9th-step. The trajectory prediction method with 1st-degree and 2nd-degree polynomial extrapolations and MTP method were attempted. The ratio of tracked marker drops remarkably at 2.5% strain in the previous MTP method. In the trajectory prediction method, the accuracy tracking is feasible up to 5% strain without dependence of deformation patterns. The 96% of successful tracking are found at the 7.5% strain, where homogeneous deformation pattern shifted to inhomogeneous one. In this case, the prediction was not success because marker position during inhomogeneous deformation was predicted by using the homogeneous deformation pattern. It is suggested that the tomography image should be scanned using a small loading step around a peak of load in experiment.

5. Application

Figure 7 shows comparison of marker tracking result in 5154 aluminum alloy. The pervious MTP method and the trajectory prediction method developed in this study were applied for this. It is found

that the trajectory prediction method increase tracked marker comparing with the MTP method. High-density tracking are achieved in the necking part of the specimen. The developed method improves the ratio of tracked marker about 10% comparing with the MTP method. Moreover, high-density tracking was also confirmed in marker tracking around a crack front in cracked sample.

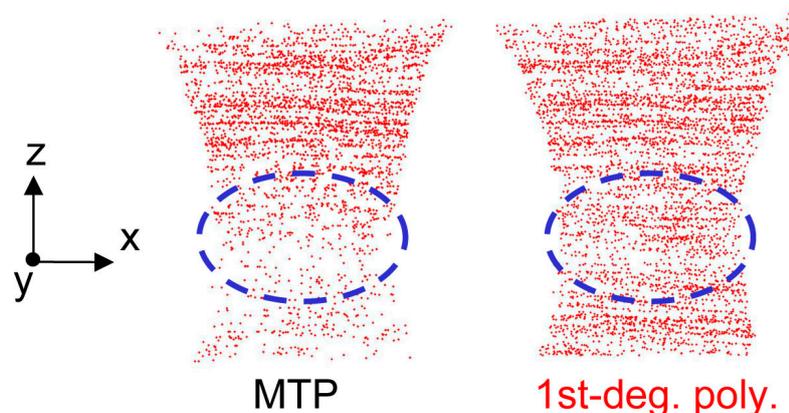


Fig. 7 Comparison of the marker tracking results in 5154 aluminum alloy.

6. Summary

The trajectory prediction method to track microstructural features in tomographic image has been developed and evaluated by using the simulated deformation model prepared in this study. The developed method with polynomial extrapolation improves ratio of tracked marker and ratio of successful tracking in case of inhomogeneous deformation developing locally. It was shown that the developed method keeps 100% of tracked marker. The high ratio of successful tracking was feasible in the range of 0-10% and 0-5% strains in the homogenous and inhomogeneous deformations, respectively. If we carry out tomographic experiments in these strain ranges, high-accurate and high-dense strain distribution without error is guaranteed by the developed method. High-dense and accurate three-dimensional strain maps can be obtained surely by applying the microstructural features tracking method developed in this study to actual deformation in aluminum alloy.

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