Proceedings of the 12th International Conference on
Aluminium Alloys, September 5-9, 2010, Yokohama, Japan

pp. 510-515

Modeling Transients Related to Strain-path Changes

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Changes of the strain-path result in transient responses of the flow stress. It is of interest to take these transients into account in constitutive models applied in finite element (FE) codes. In particular, when using explicit FE codes the extra computational cost by doing so is low. A new continuum plasticity model for strain-path changes is suggested. It is somehow similar to the Teodosiu and Hu model but is considerably simpler. It has been implemented into LS-DYNA and some preliminary results are discussed.

Keywords: *strain-path changes, metal plasticity, modeling, anisotropic materials*

1. Introduction

In the general case when following a material point history during a metal forming simulation, the strain path is changing as a function of time or strain. The strain path can change gradually, sliding along the yield surface, or in other cases abruptly, e.g. in bending-unbending of a sheet or in multi-step forming processes.

In common commercially available finite-element codes, using continuum plasticity theory as described in textbooks, e.g. [1], the aspects considered is the shape of the yield surface, its expansion as described by the isotropic hardening rule, and how it is shifted if a kinematic hardening rule is applied. The anisotropy experienced through probing proportional strain paths is given mainly by the response to the distribution of the grain orientations in the polycrystal, i.e. the measureable texture. The shape of the yield surface is commonly described by a shape function involving a limited number of parameters, which in theory can be fitted to virtual experiments calculated by a crystal plasticity model or to a range of mechanical tests. In order to change the yield locus we need to change the texture substantially. This is the case in many fabrication steps like rolling or extrusion. In forming operations on the other hand, the strain is commonly small and texture changes are not significant. It is then a reasonable approximation to keep the shape of the yield surface constant during the simulations. This corresponds to no texture evolution. For anisotropic materials the shape of the yield surface itself gives rise to a significant strain-path change contribution. This is a change of the stress level in various strain modes that is permanent as long as the new strain path is active.

In the continuum theory of plasticity it is in most cases assumed that the macroscopic plastic strain rate tensor is normal to the yield surface, i.e. the associated flow rule or simply the normality rule is adopted. The yield surface calculated by polycrystal plasticity theory, for instance by the Taylor model, may be regarded as an envelope of the yield surfaces of each grain involved. When probing proportional strain paths the normality rule is satisfied to a very good approximation. However, when the strain-path change is abrupt without unloading, which in the continuum theory of plasticity corresponds to sliding along the yield surface, normality is not obeyed. In a crystal plasticity approach using the Taylor model, Kuroda and Tvergaard [2] showed that a blunt vertex moves with the stress point along an apparently smooth yield surface. This gives rise to a transient strain-path change response lasting for less than 1-2% strain. Note that this only occurs for path changes not involving elastic unloading. The behavior for such cases is a challenge to capture by the continuum plasticity approach.

A rapid strain-path change exposes another, otherwise hidden, contribution to the anisotropic behavior, believed to arrive from the dislocation microstructures inside the grains. Typically certain dislocation structures adapt to the strain mode and form various aligned planar dense dislocation structures like cell walls or low-angle sub-boundaries, [3]. When the strain path abruptly changes, such structures cannot gradually adapt. A brand new set of slip systems may be activated that destroys the old structure and build a new one adapted to the new strain path.

The Teodosiu and Hu model, [4], is a continuum plasticity model formulated to capture the transient hardening or softening due to abrupt strain-path changes and their influence on so-called planar persistent dislocation structures. This model involves the evolution of four internal-state variables: a scalar, two second order tensors and a fourth order tensor. A consistent mathematical formulation was given later, [5]. This is a rather complex model involving many calculations due to the fourth order tensor involved. Its basic construction is to capture the transients by a rapid expansion or shrinkage of the prescribed yield locus.

In the Levkovitch model, [6], another approach is chosen adapting the shape of the yield locus as a function of the strain path. This is referred to as distortional hardening, where the distortion of the yield locus is described by the evolution of a fourth order tensor similar to the one involved in the Teodosiu and Hu model. This model is formulated for the Hill'48 yield surface. A generalization to other yield functions is non-trivial.

A detailed model for the evolution of IF-steel substructure during strain-path changes was developed by Peeters et al. [7]. This model is formulated in the crystal plasticity framework, where the critical resolved shear stresses for the different slip systems depend on the evolution of certain model parameters describing planar dislocation wall structures. It can be regarded as an advanced Taylor model.

A similar type of generalized Taylor model was proposed by Holmedal et al. [8]. This model is simpler than the Peeters model involving fewer parameters and also somehow more general in the sense that it does not try to predict specific measureable IF-steel microstructural features. The main mechanism is the hardening of non-active slips which is modeled by simple evolution equations.

The crystal plasticity models [7, 8] are very computationally demanding and not applicable in FE codes for industrial applications. The Teodosiu and Hu model is a good candidate but it is complex and involves many calculations due to the fourth order tensor involved. The idea in this paper is to develop a simpler model involving only second order tensors, and also simpler in the sense that the interpretation of its tensors is straight forward. The basic idea is to construct a second order tensor, **P**, pointing in the direction where the material has recently been strained. The direction and magnitude of **P** will act as a measure of the most recent strain history. The Bauscinger effect is modeled by an Armstrong-Fredrick type kinematic hardening rule. In addition another term takes into account the information from **P** about the recent history in order to model an initial hardening that for some materials is superposed to the Bauschinger effect.

2. The model formulation

The mechanical anisotropy of metals is commonly assumed to be controlled by the crystallographic texture and by planar dislocation structures that are formed during the most recent strain history. In the sense of continuum plasticity theory, the anisotropy resulting from the crystallographic texture is described by the yield surface. During forming operations the equivalent plastic strain remains moderate and the crystallographic rotations are sufficiently small to justify keeping the shape of this yield surface constant as a first approximation. Subsequent to abrupt or rapid strain-path changes, however, the interplay between texture and the microstructure becomes an important contribution that often cannot be neglected.

The model can be regarded as a simplified version of the Teodosiu and Hu model. The yield condition is written in the form

$$\phi \equiv \bar{\sigma}(\mathbf{T}-\mathbf{X}) - R - S_{\mathrm{o}} - S_{\mathrm{r}} = 0 \tag{1}$$

Here $\mathbf{T}$ is the Cauchy stress tensor, $\mathbf{X}$ is the back stress tensor, and S_{o} and S_{r} represent the extra strengths from directional planar dislocation structures built up during earlier straining in orthogonal and reverse directions, respectively. A function for the equivalent effective stress $\bar{\sigma}$ must be prescribed as a convex function of $(\mathbf{T}-\mathbf{X})$ describing the shape of the yield function. The model is similar to that of Teodosiu and Hu in that it expands the yield surface rapidly as a function of plastic strain subsequent to a strain-path change. Isotropic hardening R is modeled by

$$R = R_0 + \theta_{II}\varepsilon_{II}\left(1-e^{-\bar{\varepsilon}/\varepsilon_{II}}\right) + \theta_{III}\varepsilon_{III}\left(1-e^{-\bar{\varepsilon}/\varepsilon_{III}}\right) + \theta_{IV}\bar{\varepsilon} \tag{2}$$

Here $\dot{\bar{\varepsilon}}$ is the plastic strain rate, whereas R_0, θ_{II}, θ_{III} ε_{II} and ε_{III} are parameters to be fitted for the material considered.

The plastic strain rate tensor $\mathbf{D}^p$ is obtained from the associated flow rule, and thus

$$\mathbf{D}^p = \dot{\bar{\varepsilon}}\frac{\partial \bar{\sigma}}{\partial \mathbf{T}} \tag{3}$$

A second-order tensor is applied in order to capture the essence of the recent history of the strain mode. The proposed model equation makes sure that its magnitude is always smaller than unity and that it is attracted by the direction of the current strain increment, viz.

$$\dot{\mathbf{P}} = \frac{\mathbf{N}-\mathbf{P}}{\Delta\varepsilon_{\mathrm{P}}}\dot{\bar{\varepsilon}} \tag{4}$$

where $\mathbf{N} = \mathbf{D}^p / \left\|\mathbf{D}^p\right\|$. In cases of proportional straining, this model ensures that the pointer tensor $\mathbf{P}$ saturates towards the direction of the current strain increment. The parameter $\Delta\varepsilon_{\mathrm{P}}$ is a scale for how long it will take for this saturation to occur during a proportional path. The magnitude of the tensor $\left\|\mathbf{P}\right\|$ relates to the strength of the recently built up microstructure, and the direction of $\mathbf{P}$ relates to alignment of the substructure. Ideally, as is the case for the crystal plasticity approaches [7,8] this information should have been applied to distort the yield surface. This is the approach of the Levkovitch model [6]. However, this is a model that depends on a fourth order tensor and it is a challenging task to ensure convexity if a general yield surface is to be adapted. In particular this is difficult for advanced yield surfaces with a complex anisotropic shape. Instead a simpler approach is made in this paper, similar to the one by Teodosiu and Hu, who allowed shifting of the yield surface and transient expansions. The modeling of cross hardening in their model is by evolution models for a fourth order tensor and two second order tensors associated with the directional strength of planar dislocation structures and with their polarity, respectively. Furthermore the kinematic hardening was represented by a second order back-stress tensor and the isotropic hardening by a scalar.

Our approach here is simpler, where the directional strength is modeled by the two scalars S_{o} and S_{r} in addition to the pointer tensor, $\mathbf{P}$. The evolution of S_{o} relates to directional strength of planar dislocation structures, and it is modeled as:

$$\dot{S}_{\mathrm{o}} = \frac{S_0^{sat}\sqrt{\|\mathbf{P}\|^2 - |\mathbf{P}:\mathbf{N}|^2} - S_{\mathrm{o}}}{\Delta\varepsilon_{\mathrm{o}}}\dot{\bar{\varepsilon}}, \quad \text{where} \quad S_0^{sat} = q_{\mathrm{o}}R \tag{5}$$

Here q_{o} is a constant, relating the extra strength contribution S_{o}, subsequent to an orthogonal strain-path change, to the isotropic part of the stress, R. For the special case of proportional loading to a sufficiently large pre-strain, so that $\mathbf{P}$ has saturated and become a unit tensor, an abrupt strain-path change gives $\sqrt{|\mathbf{P}|^2 - |\mathbf{P}:\mathbf{N}|^2} = |\sin(\vartheta)|$. Here ϑ is the angle between the strain paths. Then, during continued loading along the new strain path, S_{o} will first saturate rapidly towards $|\sin(\vartheta)|S_0^{sat}$. The strain scale for how long this transient hardening takes to build up is proportional to $\Delta\varepsilon_{\mathrm{o}}$. Subsequent to this hardening the material will soften due to the rotation and/or change of magnitude of $\mathbf{P}$ (during a strain interval scaling with $\Delta\varepsilon_{\mathrm{p}}$), which is slower than the decrease of magnitude of S_{o}. When $\mathbf{P}$ has rotated into the same direction as $\mathbf{N}$, the contribution from S_0 vanishes, and proportional loading behavior is again the case.

Some materials exhibit an initial plateau of stress stagnation/softening subsequent to reverse strain-path changes. The directional dislocation structures (walls etc.) is here assumed to contribute to a reverse initial hardening that is superposed to the Bauschinger effect. It is assumed that the strength of this structure scales with the isotropic stress, R. This agrees with commonly assumption of scaling between the square root of the dislocation densities in dislocation wall structures and the Frank network dislocation density. This contribution is here modeled in a similar manner as the cross hardening. We define

$$\dot{S}_{\mathrm{r}} = \frac{-\mathrm{Min}\left(\mathbf{P}:\mathbf{N},0\right)q_{\mathrm{r}}R - S_{\mathrm{r}}}{\Delta\varepsilon_{\mathrm{r}}}\dot{\bar{\varepsilon}} \tag{6}$$

Say that $\mathbf{P}$ is close to $\mathbf{N}$ due to a pre-strain by a proportional strain path and next an abrupt strain-path change is made in the reverse direction. If so, $\mathbf{P}:\mathbf{N} = \|\mathbf{P}\|\cos(\vartheta)$, and all directions with $\vartheta > 90^\circ$ have a reverse element. As a consequence S_{r} will start saturating towards $\|\mathbf{P}\|q_{\mathrm{r}}R\cos(\vartheta)$, where q_{r} is a constant parameter relating this saturation value to the isotropic stress R. The structure causing this initial hardening in reverse path change will be dissolved as the reversed slip systems are activated and the parameter $\Delta\varepsilon_{\mathrm{r}}$ serves as a scale in the model equation (Eq. 6) for how long this takes. Since the build-up and dissolution of the same type of aligned dislocation structures are responsible for both cross hardening and reverse plateau type of behavior, the model parameters determining the strain interval for reverse and orthogonal hardening are assumed to be of the same order of magnitude, i.e. $\Delta\varepsilon_{\mathrm{r}} \approx \Delta\varepsilon_{\mathrm{o}}$. Eventually the material will adapt to the new mode of proportional load (the reverse of the pre-strain direction).

The back stress is modeled using a nonlinear Armstrong-Fredrick type kinematic hardening model of the form

$$\dot{\mathbf{X}} = \left(q_X R\frac{(\mathbf{T}-\mathbf{X})}{\bar{\sigma}} - \mathbf{X}\right)\frac{\dot{\bar{\varepsilon}}}{\Delta\varepsilon_{\mathrm{X}}} \tag{7}$$

The saturation value of the back stress consists of two contributions, both of them scaling with the isotropic stress, R. The strain it takes to reach this saturation level is proportional to and determined by $\Delta\varepsilon_{\mathrm{X}}$. The parameter $\Delta\varepsilon_{\mathrm{r}}$ relates to the strain scale of a sudden expansion of the yield surface when the strain path is reversed.

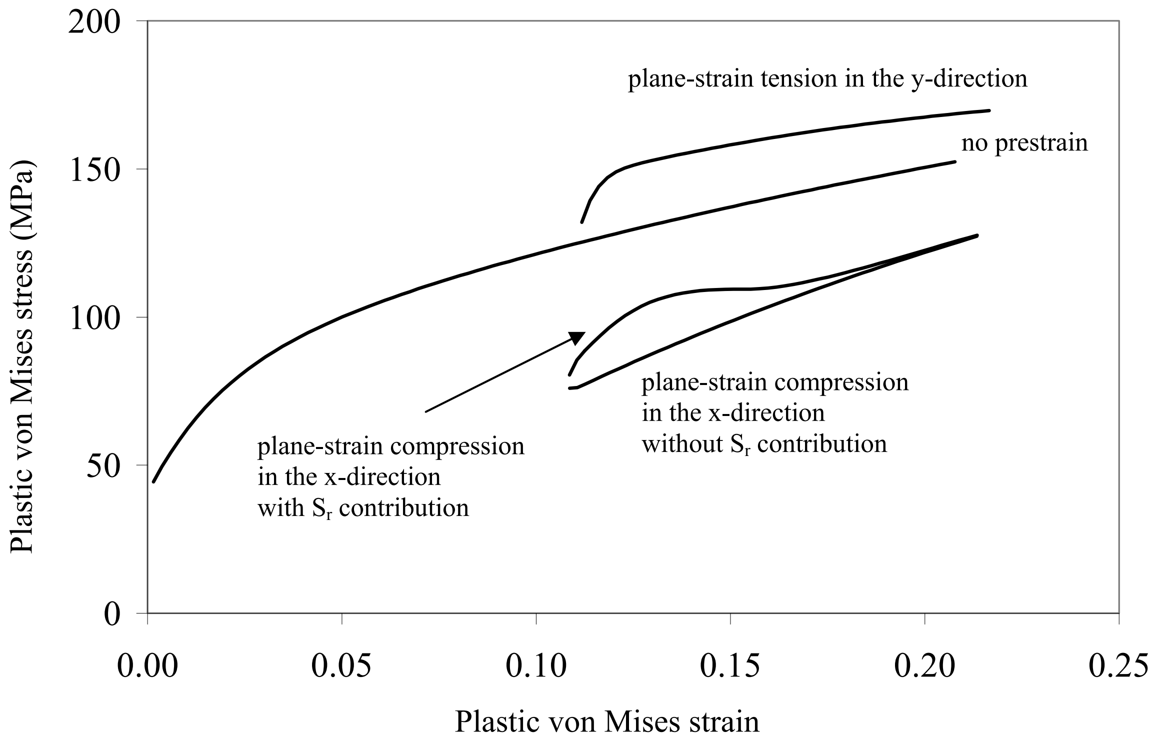

Figure 1. Von Mises stress versus plastic strain for the second step of deformation in plane strain tension in the x direction. The first step (the prestrain) had an orthogonal or reverse strain path as compared to the second step, as indicated in the figure. The strain-path change between the steps was abrupt. All curves except the reverse one indicated are without S_r contributions.

3. Results and discussion

A preliminary version of the model was implemented into LS-DYNA and some simple, single strain paths were calculated applying an isotropic von Mises yield function in Eq. 1. Fig. 1 shows the results with the following parameter selections:

$$R_0 = 40.5\,\text{MPa},\ \theta_{II} = 2308\,\text{MPa},\ \varepsilon_{II} = 0.018,\ \theta_{III} = 170\,\text{MPa},\ \varepsilon_{III} = 0.33,\ \theta_{IV} = 10.1\,\text{MPa}$$

$$\Delta\varepsilon_{\mathrm{P}} = 0.083,\ q_{\mathrm{o}} = 0.6,\ \Delta\varepsilon_0 = 0.0033,\ \Delta\varepsilon_r = 0.17,\ q_r = 0.5,\ \Delta\varepsilon_X = 0.17,\quad q_X = 0.6$$

For the sake of convenience, combinations of plane strain tension and compression in x and y directions were chosen. A prestrain with plane-strain tension in the y direction followed by an abrupt strain-path change into plane-strain tension in the x direction resulted in an orthogonal path change, i.e. the strain-rate tensor after the path change is in a direction normal to what it was during the prestrain. The result is cross-hardening by that the yield surface expands rapidly due to the increase of S_0. As the pointer tensor **P** adapts to the new strain path, the transient stress overshoot starts to decay. At strains larger than those included in the plot this curve will eventually coincide with the monotonic curve for the case with no prestrain. Note that the cross-hardening behavior is very little influenced by the kinematic hardening term **X** and by the contribution of the S_r term to the yield function.

Bauschinger type of behavior was obtained by prestrain in plane-strain compression in the x direction. When the S_r term is turned off (by setting $q_r = 0$) a classical Bauschinger effect is obtained by the kinematic hardening rule. However, when the S_r term contributes, the result is a stress plateau similar to what has been reported for experiments involving mild IF-steels or commercial purity aluminum.

4. Concluding remarks

The model qualitatively reproduces transient behaviors due to abrupt strain-path changes in a similar manner as the more complex continuum plasticity model by Teodosiu and Hu [4] and by crystal-plasticity based models [7,8]. The model can be used in combination with any yield surface and work hardening relations. This paper reports preliminary results applying an isotropic von Mises yield surface. A more detailed investigation and testing of the model are in progress and will be reported elsewhere.

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